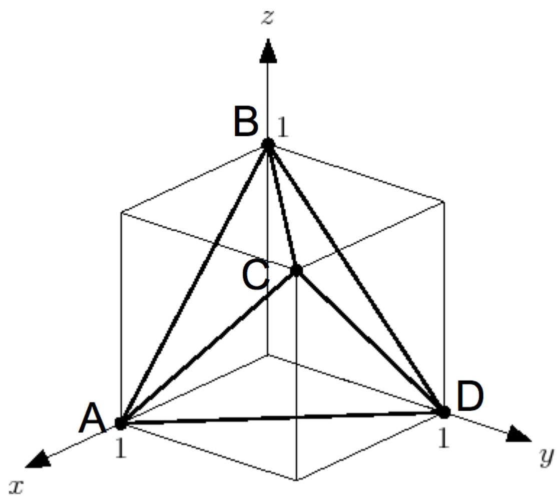
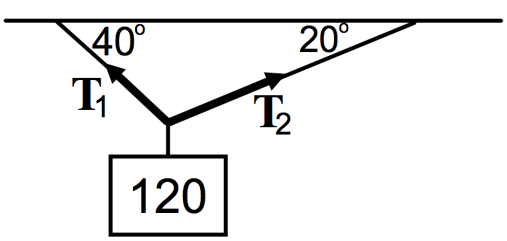
Exam 11 Name: ______________________ RHS
Multi. Calculus

Show your work for full credits

Part I (95 pts)
A unit cube is placed in the first octant, where C is at (1, 1, 1) and A, B, and D, are on x-axis, z-axis, and y-axis, respectively.



1. Evaluate $\overrightarrow{AC} \cdot \overrightarrow{AD}$.
2. What is the measure of angle CAD?
3. Find the projection vector of $\overrightarrow{AC}$ on $\overrightarrow{AD}$.
4. Find a vector that is perpendicular to both $\overrightarrow{AC}$ and $\overrightarrow{AD}$.
5. Find the area of Triangle ACD.
6. A 120-lb weight hangs from two wires as shown in the figure below. Find the tensions (forces) $\mathbf{T}_1$ and $\mathbf{T}_2$ in both wires and their magnitudes.



Part II

7. Find the volume of a parallelepiped defined by the following vectors. (10 pts)

$$\langle 2,3,5\rangle, \langle -1,3,0\rangle, \langle 5,8,0\rangle$$

Parallelepiped: a three-dimensional figure formed by six parallelograms

8. Show that the distance between a point (x_1, y_1, z_1), and a plane $Ax + By + Cz + D = 0$ is (5 pts)

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Exam 12 Name:______________________________ RHS
Multivariable Calculus

Show your work for full credits.

Part I (total 95 pts)

1. Points $A(1,2,3)$, $B(0,2,1)$, and $C(2,0,1)$ in xyz coordinate space. What is an equation of the plane that contains A, B, and C?

2. Describe the relationship between the two given planes. If they intersect, find the intersection.
$$2x + y - 4z = 9$$
$$x - y + z = 6$$

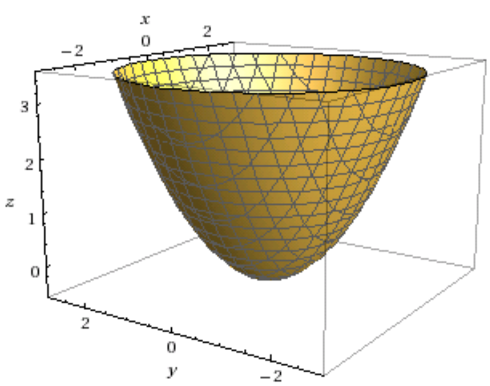
3. Describe the graph of the given vector function for $0 \le t \le 4\pi$.
$$r(t) =< t, 2t\cos t\,, t\sin t >$$

4. Match the given equations with the corresponding graphs. Explain your reasoning.

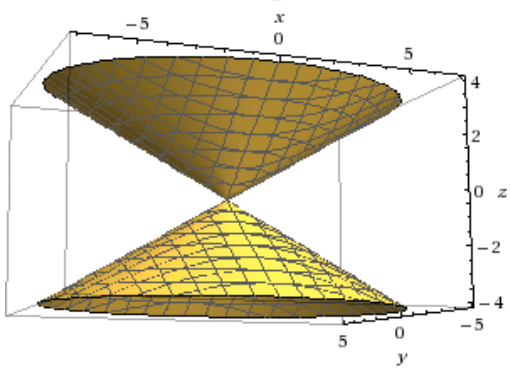
a. $x^2 + y^2 - 2z = 1$

b. $x^2 + 2y^2 - 3z^2 = 0$

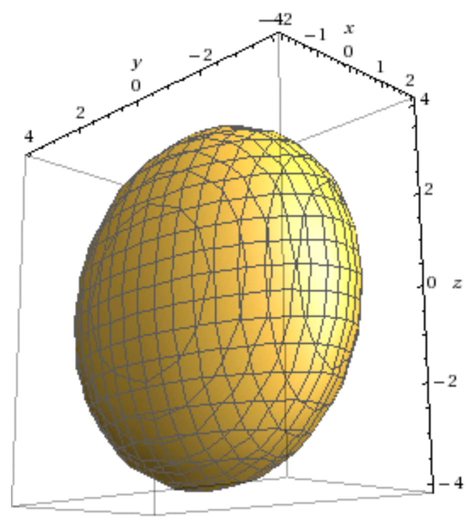
c. $4x^2 + y^2 + z^2 = 16$



i.



ii.



iii.

Part II

5. Find an equation of the tangent plane to $x^2 + y^2 + z^2 = 1$ at $\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right)$. (7 pts)

6. Find the angle formed by the two planes. (8 pts)

$$2x - y + z = 1$$
$$x + 3y - z = 1$$

Exam 13 **Name:** ______________________________ **RHS**

Multivariable Calculus

Show your work for full credits.

Part I (95 pts)

1. Find the length of the arc for $\mathbf{r}(t) =< \cos t\,, \sin t\,, \ln \cos t >, \qquad 0 \le t \le \frac{\pi}{3}$.

Let $\mathbf{r}(t) =< 2\cos t\,, 2\sin t\,, t^2 >$ for $t = 0$.

2. Find an equation of the normal plane.

3. Find the unit tangent vector at $t = 0$.

4. Find the curvature at the point $t = 0$..

Part II (15 pts)

5. Find the curvature of the curve with parametric equations.

$$x = \int_0^t \sin\left(\frac{1}{2}\pi\theta^2\right) d\theta \qquad y = \int_0^t \cos\left(\frac{1}{2}\pi\theta^2\right) d\theta$$

Exam 21 Name:______________________________ RHS

Multivariable Calculus

Show your work for full credits.

Part I (95 pts)

1. Find $\dfrac{\partial z}{\partial x}$.

$$x^2yz = \sin(x + y - z)$$

2. The radius of a right circular cone is increasing at a rate of 1 in/ s while its height is decreasing at a rate of 2 in/s. At what rate is the volume of the cone changing when the radius is 100 in. and the height is 40 in.? $V = \frac{1}{3}\pi r^2 h$

3. Verify the linear approximation at (0, 0).
$\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$

4. The wind-chill index W is the perceived temperature when the actual temperature is T and the wind speed is v, so we can write $W = f(T, v)$.

Wind speed (km/h)

Actual temperature (°C) T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

Estimate the value of $f_T(-20, 40)$ and explain the meaning of this value.

Part II (15 pts)

5. Suppose f is a differentiable function of one variable, $f(t) = \sin(t)$. Find the tangent plane to the surface $z = xf\left(\frac{y}{x}\right)$ at (1, 0). Then, approximate the value of z at (1, 0.1).

Exam 22 RHS
Multivariable Calculus

Part I (95 pts)

1. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

$$f(x,y,z) = \sqrt[3]{xyz}, \qquad (2,4,1), \quad \mathbf{v} = < 3,1,2 >$$

2. Find the local maximum and minimum values and saddle point(s) of the function.

$$f(x,y) = (x^2 + y)e^{y/2}$$

3. Find the absolute maximum and minimum values of f on the set D.

$f(x,y) = 4xy^2 - x^2y^2 - xy^3$; D is the closed triangular region in the xy-plane with vertices (0, 0), (0, 6), and (6, 0)

Part II (15 pts)

4. Among all planes that are tangent to the surface $xy^2z^2 = 1$, find the ones that are farthest from the origin.

Exam 23 Name: ____ RHS

Multi. Calc

Show your work for full credits.

1. Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.

2. Evaluate

$$\int_a^b \int_c^d 2\, dx\, dy$$

3. Evaluate

$$\int_0^1 \int_x^{e^x} 2xy^2\, dy\, dx$$

4. Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\, dx\, dy$$

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5. State if the given statement is true or false. Explain without using computations from calculator.

$$\int_1^4 \int_0^1 \left(x^2 + \sqrt{y}\right) \sin(x^2y^2)\, dx\, dy \leqslant 9$$